NUMERICAL SOLUTION OF PROBLEMS OF THE FILTRATION OF A LIQUID IN ELASTICALLY DEFORMABLE CRACKED-POROUS MEDIA

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Results are presented from a numerical study of problems concerning filtration in cracked-porous reservoirs whose exploitation is accompanied by substantial deformation of the main flow channels (cracks).

Field observations and experiments show that the characteristics of filtration processes in cracked-porous reservoirs are heavily nonlinearly dependent on the reservoirs' stress state. The main features of the operative mechanism here are satisfactorily described in dimensionless form by the equations [1]:

$$a\frac{\partial p_1}{\partial t} = g\nabla(p_1^3\nabla p_1) + p_2 - p_1, \quad \frac{\partial p_2}{\partial t} = \varepsilon g\Delta p_2 - p_2 + p_1, \tag{1}$$

where $\varepsilon \ll 1$. The dimensionless characteristics in (1) are chosen so that the parameter $g = 1$ for an infinite region and $g < 1$ for a finite region. Since no exact solutions of (1) are known to exist, it is important to have results from numerical studies of different boundary-value problems for (1). Such results are presented below. An approximate analysis of filtration processes within the framework of the present approach was made in [2, 3].

1. With allowance for the recommendations in [4] on the approximation of a quasilinear heat-conduction equation (uniform grid, order of approximation $O(h^2 + \tau)$), difference analog (1) has the following form in the case of cylindrical symmetry:

$$\begin{gathered}
a\frac{\hat{u}_i - u_i}{\tau} = \frac{g}{r_i h}\left(r_{i+1/2}e_{i+1}\frac{\hat{u}_{i+1} - \hat{u}_i}{h} - r_{i-1/2}e_i\frac{\hat{u}_i - \hat{u}_{i-1}}{h}\right) + v_i - u_i,\\
\frac{\hat{v}_i - v_i}{\tau} = \varepsilon g\left(\frac{1}{r_i}\frac{\hat{v}_{i+1} - \hat{v}_{i-1}}{2h} + \frac{\hat{v}_{i+1} - 2\hat{v}_i + \hat{v}_{i-1}}{h^2}\right) - v_i + u_i,\\
e_i = (u_i^3 + u_{i-1}^3)/2, \; r_{i\pm 1/2} = \left(i \pm \frac{1}{2}\right)h \; (i = 1, \ldots, n-1).
\end{gathered} \tag{2}$$

Using representations (2) in standard form

$$\begin{gathered}
A_i\hat{u}_{i-1} + B_i\hat{u}_i + C_i\hat{u}_{i+1} = D_i,\\
L_i\hat{v}_{i-1} + M_i\hat{v}_i + N_i\hat{v}_{i+1} = K_i,
\end{gathered} \tag{3}$$

where the coefficients

$$\begin{gathered}
A_i = -WXe_i, \; C_i = -WYe_{i+1}, \; B_i = a/\tau - A_i - C_i,\\
D_i = v_i + (a/\tau - 1)u_i, \; L_i = -\varepsilon WX,\\
M_i = 2\varepsilon g/h^2 + 1/\tau, \; K_i = (1/\tau - 1)v_i + u_i,\\
N_i = -\varepsilon WY, \; W = g/(2ih^2), \; X = 2i - 1, \; Y = 2i + 1
\end{gathered}$$

are determined from the previous time layer, we obtain the trial-run formulas:

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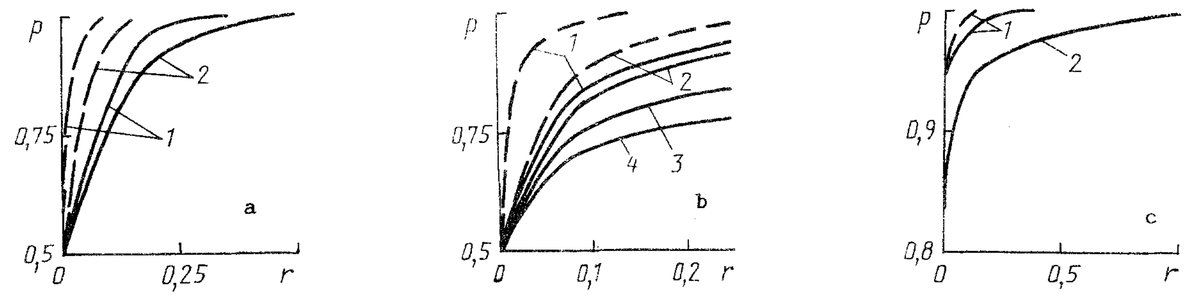

Fig. 1. Distribution of pressure in the reservoir (p_1 — solid curves; p_2 — dashed curves): a) infinite bed, $p_0 = 0.5$, $a = 1$, $t_1 = 0.01$ (1), $t_2 = 0.1$ (2); b) closed bed, $p_0 = 0.5$, $a = 0.1$, $t_1 = 0.1$ (1), $t_2 = 1$ (2), $t_3 = 10$ (3), $t_4 = 20$ (4) (at $t \geq t_3$, $p_1 \approx p_2$); c) open bed, $q = 10^{-4}$, $a = 1$, $t_1 = 0.1$ (1); $t_2 = 50$ (2) (at $t \geq t_2$, $p_1 \approx p_2$).

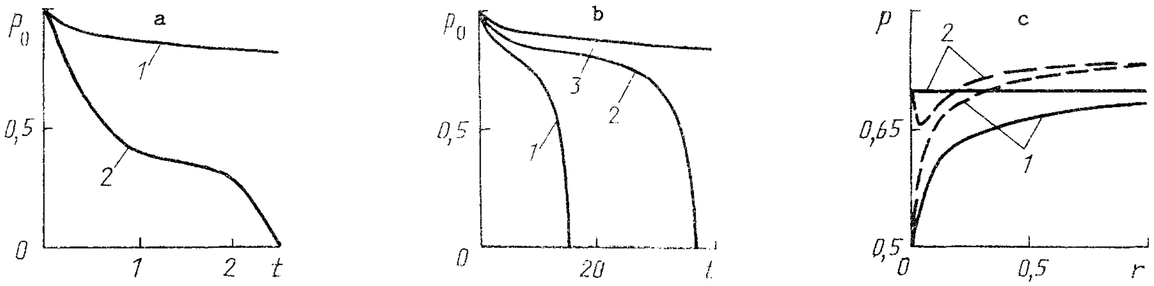

Fig. 2. Drop of bottom-hole pressure (a, b) and pressure recovery (c): a) open bed, $q = 10^{-4}$ (1), 10^{-3} (2); b) closed bed, $q = 10^{-4}$, $a = 0.1$ (1), 1 (2), 10 (3); c) closed bed (p_1 — solid curves; p_2 — dashed curves), moment of closure of well $T = 2$ (1), $t = T + 0.1$ (2).

$$
\begin{gathered}
\hat{u}_i = F_i\hat{u}_{i+1} + H_i, \quad \hat{v}_i = R_i\hat{v}_{i+1} + S_i, \\
F_i = -C_i/E_i, \quad H_i = \alpha_i/E_i, \quad E_i = B_i + A_iF_{i-1}, \\
R_i = -N_i/Z_i, \quad S_i = \beta_i/Z_i, \quad Z_i = M_i + L_iR_{i-1}, \\
\alpha_i = D_i - A_iH_{i-1}, \quad \beta_i = K_i - L_iS_{i-1} \quad (i = 1, \ldots, n-1). \\
F_0 = R_0 = 0, \quad H_0 = \hat{u}_0, \quad S_0 = \hat{v}_0.
\end{gathered} \tag{4}
$$

We will examine basic boundary-value problems: either a constant pressure or a constant yield is assigned for the well; the bed is infinite or finite and open or closed (accordingly, $p_1 = p_2 = 1$ or $\partial p_1/\partial r = \partial p_2/\partial r = 0$ at $r = 1$); pressure recovery problem.

In assigning the yield of the well

$$
q = r_0\left(p_1^3\,\frac{\partial p_1}{\partial r} + \varepsilon\,\frac{\partial p_2}{\partial r}\right)
$$

we obtain the following for the grid functions

$$
\hat{u}_0 = \hat{v}_0 = (u_0^3\hat{u}_1 + \varepsilon\hat{v}_1 - hq/r_0)/(u_0^3 + \varepsilon).
$$

In this case, as follows from (3), the trial-run formulas have the following form instead of (4)

$$
\begin{gathered}
\hat{u}_i = F_i\hat{u}_{i+1} + G_i\hat{v}_{i+1} + H_i, \quad \hat{v}_i = P_i\hat{u}_{i+1} + R_i\hat{v}_{i+1} + S_i, \\
F_i = -\gamma_iC_i/\varkappa_i, \quad G_i = \delta_iN_i/\varkappa_i, \quad H_i = (\alpha_i\gamma_i - \beta_i\delta_i)/\varkappa_i, \\
P_i = \omega_iC_i/\varkappa_i, \quad R_i = -\nu_iN_i/\varkappa_i, \quad S_i = (\beta_i\nu_i - \alpha_i\omega_i)/\varkappa_i,
\end{gathered}
$$

$$
\begin{gathered}
\gamma_i = L_i R_{i-1} + M_i,\ \delta_i = A_i G_{i-1},\ \omega_i = L_i P_{i-1}, \\
\nu_i = A_i F_{i-1} + B_i,\ \varkappa_i = \gamma_i \nu_i - \delta_i \omega_i\ (i = 1,\ \ldots,\ n-1), \\
F_0 = P_0 = u_0^3/l,\ G_0 = R_0 = \varepsilon/l,\ H_0 = S_0 = hq/(rl), \\
l = u_0^3 + \varepsilon.
\end{gathered} \tag{5}
$$

For a closed bed, we find from (5) that

$$
\begin{gathered}
\hat{u}_n = \hat{u}_{n-1} = k_1/m,\ \hat{v}_n = \hat{v}_{n-1} = k_2/m, \\
k_1 = S_j G_j - H_j (R_j - 1),\ k_2 = H_j P_j - S_j (F_j - 1), \\
m = (F_j - 1)(R_j - 1) - P_j G_j,\ j = n - 1.
\end{gathered}
$$

The parameter a in (1) can have any value [1]. Numerical calculations were performed for $a = 0.1, 1, 10$, $\varepsilon = 0.01$, $g = 1, 0.1$. We chose the characteristic time for a cracked-porous medium as the time scale in (1). It was therefore interesting to obtain results for $t \ll 1$, $t \sim 1$, $t \gg 1$. In the dimensionless variables used here, the yield $q < 1$, $r_0 \leq r \leq 1$.

2. An analysis of the numerical solutions — some of which are shown in Figs. 1 and 2 — showed their qualitative agreement with the approximate results in [2, 3]. The corresponding quantitative deviations in the variants chosen for comparison did not exceed 30%.

Let us present the main conclusions from the numerical study of (1).

The pressure curves $p_1 = p_1(r)$, $p_2 = p_2(r)$ are characterized by a steep initial section (which is steeper in the blocks). During the beginning of operation of the well, it is mainly the cracks that react first ($t \ll 1$); the perturbation zone develops considerably more rapidly in the cracks than in the blocks (Fig. 1a — dimensionless parameters are used in the figures).

Different regimes of reservoir operation are possible as time passes.

When a constant bottom-hole pressure (open or closed region) or a constant yield $q < q^*$ (open region, q^* being a certain critical yield) is maintained, the propagation of a disturbance in the cracks is slowed and the pressure curves p_1 and p_2 approach one another. At $t \gg 1$, the reservoir operates as though it were uniform (Fig. 1b and c). It should be noted that compared to the linear model [5] of a cracked-porous medium (see [6] for a qualitative analysis), in the case of Eqs. (1) there cannot be a substantial increase in perturbation in the blocks over time compared to the situation in the cracks.

With a constant yield $q \geq q^*$ in the case of an open region or any value of q for a closed region, pressure decreases in the cracks faster over time than in the blocks. Bottom-hole pressure decreases sharply as the critical pressure $p = 0$ is approached (this is the pressure at which the cracks are closed up). In this case, the curves $p_0 = p_0(t)$ have a distinct "two-layer" appearance (Fig. 2a and b).

In the conditions corresponding to Fig. 2a (open bed) for $q = 10^{-4}$, a constant uniform pressure distribution $p_1 \approx p_2$ (curve 1) is established by the moment $t = 19.9$ ($a = 0.1$), $t = 50$ ($a = 1$), $t = 218$ ($a = 10$) and the cracks do not close. For $a = 1$ and $q = 10^{-3}$ (curve 2), a crack-closure front forms at the moment $t = 2$. In the case of a closed bed (Fig. 2b) for $q = 10^{-4}$, crack closure occurs at the moment $t = 16.3$ ($a = 0.1$), $t = 38$ ($a = 1$), $t = 175$ ($a = 10$).

It follows from the numerical results that an increase in the parameter a and a reduction in depression slow filtration processes in the bed.

Estimates of q^* are presented in [3].

With closure of the well, recovery of pressure occurs first in the cracks. Pressure recovery is retarded significantly in the blocks. In the numerical example (Fig. 2c), the reservoir was operating as a two-layer medium by the moment of closure ($T = 2$). For $t \geq T + 0.1$, the pressure distribution in the cracks is nearly uniform, $p_1 = p_1(t)$. The blocks are fed by cracks away from the well, but to a lesser degree than before closure. Reverse flow takes place near the well, since there has been sufficient time for the occurrence of pressure recovery in the cracks.

NOTATION

Here p, r, t, are pressure, coordinate, time; h, τ, coordinate and time steps; u, v, grid functions; q, well yield; r_0, radius of well; a, ε, g, system parameters; a superimposed circumflex denotes values of the grid functions for the next time layer.

LITERATURE CITED

1. Yu. A. Buevich, Inzh.-Fiz. Zh., **46**, No. 4, 593-600 (1984).
2. V. S. Nustrov, Inzh.-Fiz. Zh., **53**, No. 4, 617-624 (1987).
3. V. S. Nustrov, Inzh.-Fiz. Zh., **53**, No. 6, 994-1000 (1987).
4. A. A. Samarskii, Theory of Difference Schemes [in Russian], Moscow (1983).
5. G. I. Barenblatt, V. M. Entov, and V. M. Ryzhik, Theory of the Unsteady Filtration of Liquids and Gases [in Russian], Moscow (1972).
6. V. N. Nikolaevskii, K. S. Basniev, A. T. Borbunov, and G. A. Zotov, Mechanics of Saturated Porous Media [in Russian], Moscow (1970).

DESCRIPTION OF THE STRENGTH OF POROUS BODIES ON THE BASIS OF PERCOLATION THEORY

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A method is proposed for estimating the strength of porous materials in destructive rupture on the basis of the percolation theory of regularly packed spheres. The results of calculation by this method are in good agreement with experimental data in the whole porosity range of the material.

Percolation theory [1, 2] is widely used to describe various probabilistic processes. There have been numerous studies of the elasticity of two- and three-dimensional percolation systems, for example [3-7]; it was noted in [4, 5] that the elasticity and conductivity problems for the volume elastic modulus of gel at the gel point belong to different universality classes. In [7], the method of reduction to an elementary cell was used to determine the moduli of the percolation systems, permitting considerable simplification of the calculations. An analogy between the mechanical characteristics and thermal conductivity of porous powder materials was made in [8]. In [9], percolation theory was used to describe the strength and rheological characteristics of disperse systems; however, the use of Bethe lattices limits the application of the given approach to transitions of sol-gel type. The percolational approach to the description of the strength of porous media was considered in [10]; the dependence of the relative strength G (short-term resistance of the material in rupture referred to its maximum value) on the relative particle concentration $m/m_0 = (1 - \Pi)/(1 - \Pi_0)$ where Π_0 is the porosity of the system corresponding to the densest random packing of the particles, was derived. One advantage of this approach is the elimination of the traditional consideration of the strength using the concept of the "failure surface," which leads to complication in taking account of statistical inhomogeneities of the system, for example, macropores. The calculation method is based on an analogy between conduction and the strength of a percolation system, but this analogy is more explicit if the proportion of conducting point of the percolational system is identified with the mean relative coordination number $\nu = (Z + 1)/(Z_0 + 1)$, which characterizes the proportion of possible particle bonds realized on average in the given case.

An analytical expression from [11] is used to calculate the mean coordination number of a lattice of randomly packed spherical particles as a function of the porosity of the packing

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